# COMPLEX NUMBER

## 1. DEFINITION

A number of the form a + ib, where a, b ∈ R and $i = \sqrt{-1}$, is called a complex number and is denoted by 'Z'.

$$\boxed{z = \boxed{a} + i\boxed{b}}$$

$$\qquad\downarrow \qquad\quad \downarrow$$

$$\text{Re}(z) \quad \text{Im}(z)$$

### 1.1 Conjugate of a Complex Number

For a given complex number z = a + ib,

its conjugate '$\bar{z}$' is defined as $\bar{z}$ = a – ib

## 2. ALGEBRA OF COMPLEX NUMBERS

Let $z_1$ = a + ib and $z_2$ = c + id be two complex numbers where a, b, c, d ∈ R and $i = \sqrt{-1}$.

**1. Addition :**

$z_1 + z_2$ = (a + bi) + (c + di)

= (a + c) + (b + d) i

**2. Subtraction :**

$z_1 - z_2$ = (a + bi) – (c + di)

= (a – c) + (b – d) i

**3. Multiplication :**

$z_1 \cdot z_2$ = (a + bi) (c + di)

= a (c + di) + bi (c + di)

= ac + adi + bci + bdi$^2$

= ac – bd + (ad + bc) i

($\because i^2 = -1$)

**4. Division :**

$$\frac{z_1}{z_2} = \frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di}$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$$

*Note...*

1. a + ib = c + id

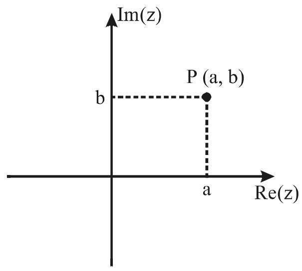
⇔ a = c & b = d

2. $$i^{4k+r} = \begin{cases} 1; & r = 0 \\ i; & r = 1 \\ -1; & r = 2 \\ -i; & r = 3 \end{cases}$$

3. $\sqrt{`}\,\sqrt{a} = \sqrt{`\,a}$ only if atleast one of either a or b is non-negative.

## 3. ARGAND PLANE

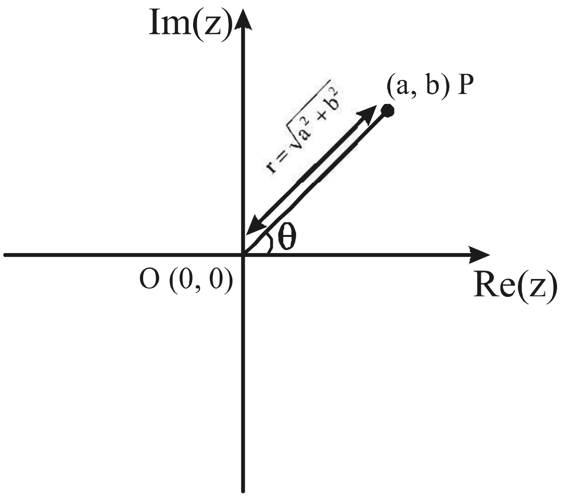
A complex number z = a + ib can be represented by a unique point P (a, b) in the argand plane.




Z = a + ib is represented by a point P (a, b)

## 3.1 Modulus and Argument of Complex Number

If z = a + ib is a complex number




(i) Distance of Z from origin is called as modulus of complex number Z.

It is denoted by $r=|z|=\sqrt{a^2+b^2}$

(ii) Here, θ i.e. angle made by OP with positive direction of real axis is called **argument of z**.

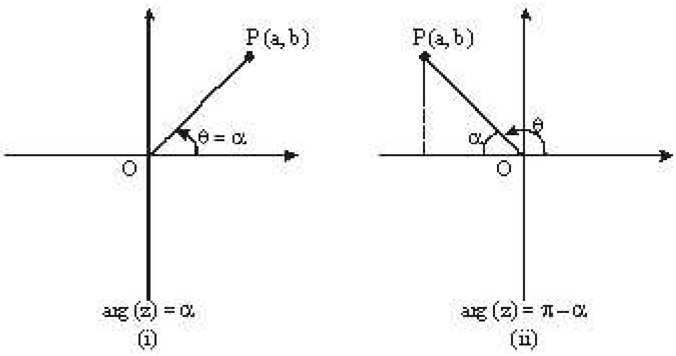
*Note...*

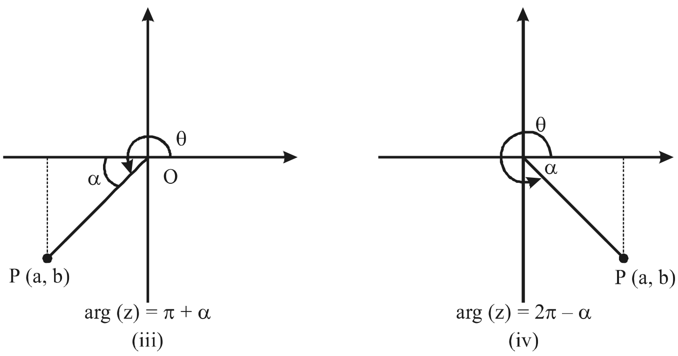
$z_1 > z_2$ or $z_1 < z_2$ has no meaning but $|z_1| > |z_2|$ or $|z_1| < |z_2|$ holds meaning.

## 3.2 Principal Argument

The argument 'θ' of complex number z = a + ib is called principal argument of z if $-\pi < \theta \le \pi$.

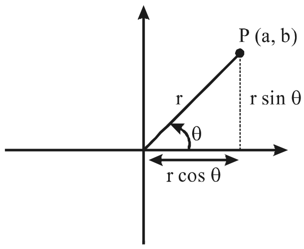
Let $\tan\alpha = \left|\frac{b}{a}\right|$, and θ be the arg (z).




In (iii) and (iv) principal argument is given by $-\pi + \alpha$ and $-\alpha$ respectively.

## 4. POLAR FORM




$a = r\cos\theta \quad \& \; b = r\sin\theta;$

where $r = |z|$ and $\theta = \arg(z)$

$\therefore \quad z = a + ib$

$\qquad = r(\cos\theta + i\sin\theta)$

*Note...*

$Z = re^{i\theta}$ is known as Euler's form; where $r = |Z|$ & $\theta = \arg(Z)$

## 5. SOME IMPORTANT PROPERTIES

**1.** $\overline{(\overline{z})} = z$

**2.** $z + \overline{z} = 2\mathrm{Re}(z)$

**3.** $z - \overline{z} = 2i\,\mathrm{Im}(z)$

**4.** $\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$

**5.** $\overline{z_1 z_2} = \overline{z}_1\,\overline{z}_2$

**6.** $|z| = 0 \Rightarrow z = 0$

**7.** $z\overline{z} = |z|^2$

**8.** $|z_1 z_2| = |z_1||z_2|$; $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

**9.** $|\overline{z}| = |z| = |-z|$

**10.** $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\,\mathrm{Re}\,(z_1\,\overline{z}_2)$

**11.** $|z_1 + z_2| \le |z_1| + |z_2|$ (Triangle Inequality)

**12.** $|z_1 - z_2| \ge ||z_1| - |z_2||$

**13.** $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

**14.** $\mathrm{amp}\,(z_1 . z_2) = \mathrm{amp}\; z_1 + \mathrm{amp}\; z_2 + 2k\pi\,;\, k \in I$

**15.** $\mathrm{amp}\left(\frac{y_0}{y_1}\right) = \mathrm{amp}\; z_1 - \mathrm{amp}\; z_2 + 2k\pi\;;\, k \in I$

**16.** $\mathrm{amp}(z^n) = n\,\mathrm{amp}(z) + 2k\pi\,;\, k \in I$

## 6. DE-MOIVRE'S THEOREM

**Statement :** $\cos n\theta + i\sin n\theta$ is the value or one of the values of $(\cos\theta + i\sin\theta)^n$ according as if 'n' is integer or a rational number. The theorem is very useful in determining the roots of any complex quantity

## 7. CUBE ROOT OF UNITY

Roots of the equation $x^3 = 1$ are called cube roots of unity.

$$x^3 - 1 = 0$$

$$(x - 1)(x^2 + x + 1) = 0$$

$$x = 1 \quad \text{or} \quad x^2 + x + 1 = 0$$

$$\text{i.e} \quad x = \underbrace{\frac{-1+\sqrt{3}i}{2}}_{w} \quad \text{or} \quad x = \underbrace{\frac{-1-\sqrt{3}i}{2}}_{w^2}$$

**(i)** The cube roots of unity are $1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$.

**(ii)** $W^3 = 1$

**(iii)** If w is one of the imaginary cube roots of unity then $1 + w + w^2 = 0$.

**(iv)** In general $1 + w^r + w^{2r} = 0$ ; where $r \in I$ but is not the multiple of 3.

**(v)** In polar form the cube roots of unity are :

$$\cos 0 + i\sin 0\,;\; \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3},\; \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

**(vi)** The three cube roots of unity when plotted on the argand plane constitute the verties of an equilateral triangle.

**(vii)** The following factorisation should be remembered :

$a^3 - b^3 = (a - b)(a - \omega b)(a - \omega^2 b)$ ;

$x^2 + x + 1 = (x - \omega)(x - \omega^2)$ ;

$a^3 + b^3 = (a + b)(a + \omega b)(a + \omega^2 b)$;

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a + \omega b + \omega^2 c)(a + \omega^2 b + \omega c)$

## 8. 'n' n[th] ROOTS OF UNITY

Solution of equation $x^n = 1$ is given by

$$x = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} \quad ; k = 0, 1, 2, ..., n-1$$

$$= e^{i\left(\frac{2k\pi}{n}\right)} \quad ; k = 0, 1, ....., n-1$$

*Note...*

1. We may take any n consecutive integral values of k to get 'n' n[th] roots of unity.
2. Sum of 'n' n[th] roots of unity is zero, $n \in N$
3. The points represented by 'n' n[th] roots of unity are located at the vertices of regular polygon of n sides inscribed in a unit circle, centred at origin & one vertex being one +ve real axis.

**Properties :**

If $1, \alpha_1, \alpha_2, \alpha_3 ..... \alpha_{n-1}$ are the n, n[th] root of unity then :

**(i)** They are in G.P. with common ratio $e^{i(2\pi/n)}$

**(ii)** $1^p + \alpha_1^p + \alpha_2^p + .... + \alpha_{n-1}^p = \begin{cases} 0, & \text{if } p \neq kn \\ n, & \text{if } p = kn \end{cases}$ where $k \in Z$

**(iii)** $(1-\alpha_1)(1-\alpha_2) ...... (1-\alpha_{n-1}) = n$

**(iv)** $(1+\alpha_1)(1+\alpha_2) ....... (1+\alpha_{n-1}) = \begin{cases} 0, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$

**(v)** $1 . \alpha_1 . \alpha_2 . \alpha_3 ......... \alpha_{n-1} = \begin{cases} -1, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$

*Note...*

**(i)** $\cos\theta + \cos 2\theta + \cos 3\theta + ..... + \cos n\theta = \frac{\sin(n\theta/2)}{\sin(\theta/2)} \cos\left(\frac{n+1}{2}\right)\theta.$

**(ii)** $\sin\theta + \sin 2\theta + \sin 3\theta + ..... + \sin n\theta = \frac{\sin(n\theta/2)}{\sin(\theta/2)} \sin\left(\frac{n+1}{2}\right)\theta.$

## 9. SQUARE ROOT OF COMPLEX NUMBER

Let $x + iy = \sqrt{a + ib}$, Squaring both sides, we get

$(x + iy)^2 = a + ib$

i.e. $x^2 - y^2 = a,\ 2xy = b$

Solving these equations, we get square roots of z.

## 10. LOCI IN COMPLEX PLANE

(i) $|z - z_o| = a$ represents circumference of circle, centred at $z_o$, radius a.

(ii) $|z - z_o| < a$ represents interior of circle

(iii) $|z - z_o| > a$ represents exterior of this circle.

(iv) $|z - z_1| = |z - z_2|$ represents $\perp$ bisector of segment with end points $z_1$ & $z_2$.

(v) $\left|\frac{z - z_1}{z - z_2}\right| = k$ represents : $\begin{Bmatrix} \text{circle}, & k \neq 1 \\ \perp \text{bisector}, & k = 1 \end{Bmatrix}$

(vi) arg (z) = θ is a ray starting from origin (excluded) inclined at an $\angle\theta$ with real axis.

(vii) Circle described on line segment joining $z_1$ & $z_2$ as diameter is :

$$(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0.$$

(viii) Four pts. $z_1, z_2, z_3, z_4$ in anticlockwise order will be concyclic, if & only if

$$\theta = \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$$

$$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) - \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right) = 2n\pi \ ; (n \in I)$$

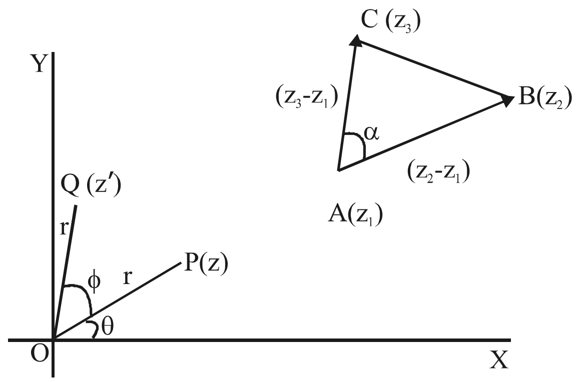
$$\Rightarrow \arg\left[\left(\frac{z_2 - z_4}{z_1 - z_4}\right)\left(\frac{z_1 - z_3}{z_2 - z_3}\right)\right] = 2n\pi$$

$$\Rightarrow \left(\frac{z_2 - z_4}{z_1 - z_4}\right) \times \left(\frac{z_1 - z_3}{z_2 - z_3}\right) \text{ is real \& positive.}$$

## 11. VECTORIAL REPRESENTATION OF A COMPLEX

Every complex number can be considered as if it is the position vector of that point. If the point P represents the complex number z then,

$$\vec{OP} = z \quad \& \quad |\vec{OP}| = |z|.$$




*Note...*

**(i)** If $\vec{OP} = z = r e^{i\theta}$ then $\vec{OQ} = z_1 = r e^{i(\theta+\phi)} = z \,.\, e^{i\phi}$.

If $\vec{OP}$ and $\vec{OQ}$ are of unequal magnitude then

$$\hat{OQ} = \hat{OP}\, e^{i\phi}$$

**(ii)** If $z_1$, $z_2$, $z_3$, are three vertices of a triangle ABC described in the counter-clock wise sense, then

$$\frac{z_3 - z}{z_2 - z} = \frac{AC}{AB}(\cos\alpha + i\sin\alpha) = \frac{AC}{AB}.e^{i\alpha} = \frac{|z_3 - z_1|}{|z_2 - z_1|}.e^{i\alpha}$$

## 12. SOME IMPORTANT RESULTS

(i) If $z_1$ and $z_2$ are two complex numbers, then the distance between $z_1$ and $z_2$ is $|z_2 - z_1|$.

(ii) Segment Joining points A ($z_1$) and B($z_2$) is divided by point P (z) in the ratio $m_1 : m_2$

then $z = \dfrac{m_1 z_2 + m_2 z}{m_1 + m_2}$, $m_1$ and $m_2$ are real.

(iii) The equation of the line joining $z_1$ and $z_2$ is given by

$$\begin{vmatrix} z & \bar{z} & \\ z & \bar{z} & \\ z_2 & \bar{z}_2 & \end{vmatrix} = 0 \text{ (non parametric form)}$$

Or

$$\frac{z - z}{\bar{z} - \bar{z}} = \frac{z - z_2}{\bar{z} - \bar{z}_2}$$

(iv) $\bar{a}z + a\bar{z} + b = 0$ represents general form of line.

(v) The general eqn. of circle is :

$$z\bar{z} + a\bar{z} + \bar{a}z + b = 0 \qquad \text{(where b is real no.).}$$

Centre : (–a) & radius $\sqrt{|a|^2 - b} = \sqrt{a\bar{a} - b}$.

(vi) Circle described on line segment joining $z_1$ & $z_2$ as diameter is :

$$(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0.$$

(vii) Four pts. $z_1, z_2, z_3, z_4$ in anticlockwise order will be concylic, if & only if

$$\theta = \arg.\left(\frac{z_2 - z_4}{z_1 - z_4}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$$

$$\Rightarrow \quad \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) - \arg.\left(\frac{z_2 - z_3}{z_1 - z_3}\right) = 2n\pi \; ; (n \in I)$$

$$\Rightarrow \quad \arg\left[\left(\frac{z_2 - z_4}{z_1 - z_4}\right)\left(\frac{z_1 - z_3}{z_2 - z_3}\right)\right] = 2n\pi$$

$$\Rightarrow \quad \left(\frac{z_2 - z_4}{z_1 - z_4}\right) \times \left(\frac{z_1 - z_3}{z_2 - z_3}\right) \text{ is real \& positive.}$$

(viii) If $z_1, z_2, z_3$ are the vertices of an equilateral triangle where $z_0$ is its circumcentre then

**(a)** $\frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2} = 0$

**(b)** $z_0^1 + z_1^1 + z_2^1 - z_1 z_2 - z_2 z_3 - z_3 z_1 = 0$

**(c)** $z_0^1 + z_1^1 + z_2^1 = 3\,z_/^1$

(ix) If A, B, C & D are four points representing the complex numbers $z_1, z_2, z_3$ & $z_4$ then

AB | | CD if $\frac{z_4 - z_3}{z_2 - z_1}$ is purely real ;

AB $\perp$ CD if $\frac{z_4 - z_3}{z_2 - z_1}$ is purely imaginary ]

(x) Two points P $(z_1)$ and Q$(z_2)$ lie on the same side or opposite side of the line $\bar{a}z + a\bar{z} + b$ accordingly as $\bar{a}z_1 + a\bar{z}_1 + b$ and $\bar{a}z_2 + a\bar{z}_2 + b$ have same sign or opposite sign.

**Important Identities**

(i) $x^2 + x + 1 = (x-\omega)(x-\omega^2)$

(ii) $x^2 - x + 1 = (x + \omega)(x + \omega^2)$

(iii) $x^2 + xy + y^2 = (x-y\omega)(x-y\omega^2)$

(iv) $x^2 - xy + y^2 = (x + \omega y)(x + y\omega^2)$

(v) $x^2 + y^2 = (x + iy)(x - iy)$

(vi) $x^3 + y^3 = (x + y)(x + y\omega)(x + y\omega^2)$

(vii) $x^3 - y^3 = (x - y)(x - y\omega)(x - y\omega^2)$

(viii) $x^2 + y^2 + z^2 - xy - yz - zx = (x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$

or $(x\omega + y\omega^2 + z)(x\omega^2 + y\omega + z)$

or $(x\omega + y + z\omega^2)(x\omega^2 + y + z\omega)$.

(ix) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x+\omega y+\omega^2 z)(x + \omega^2 y + \omega z)$

# QUADRATIC EQUATION

## 1. QUADRATIC EXPRESSION

The general form of a quadratic expression in x is,

$f(x) = ax^2 + bx + c$, where a, b, c $\in$ R & $a \neq 0$.

and general form of a quadratic equation in x is,

$ax^2 + bx + c = 0$, where a, b, c $\in$ R & $a \neq 0$.

## 2. ROOTS OF QUADRATIC EQUATION

**(a) The solution of the quadratic equation,**

$ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $D = b^2 - 4ac$ is called the discriminant of the quadratic equation.

**(b) If $\alpha$ & $\beta$ are the roots of the quadratic equation**

$ax^2 + bx + c = 0$, then ;

(i) $\alpha + \beta = -b/a$ (ii) $\alpha\beta = c/a$

(iii) $|\alpha - \beta| = \dfrac{\sqrt{D}}{|a|}$.

**(c) A quadratic equation whose roots are $\alpha$ & $\beta$ is $(x - \alpha)(x - \beta) = 0$ i.e.**

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e.

$x^2$ – (sum of roots) x + product of roots = 0.

*Note...*

$$y = (ax^2 + bx + c) \equiv a(x - \alpha)(x - \beta)$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a}$$

## 3. NATURE OF ROOTS

**(a) Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ R & $a \neq 0$ then;**

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).

(iii) $D < 0 \Leftrightarrow$ roots are imaginary.

(iv) If p + i q is one root of a quadratic equation, then the other must be the conjugate p – i q & vice versa. $\left(p,\ q \in R\ \&\ i = \sqrt{-1}\right)$.

**(b) Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ Q & $a \neq 0$ then;**

(i) If D > 0 & is a perfect square, then roots are rational & unequal.

(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

*Note...*

Remember that a quadratic equation cannot have three different roots & if it has, it becomes an identity.

## 4. GRAPH OF QUADRATIC EXPRESSION

Consider the quadratic expression, $y = ax^2 + bx + c$, $a \neq 0$ & $a, b, c \in R$ then ;

(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards & if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $y > 0 \ \forall \ x \in R$, only if $a > 0$ & $D < 0$

(iii) $y < 0 \ \forall \ x \in R$, only if $a < 0$ & $D < 0$

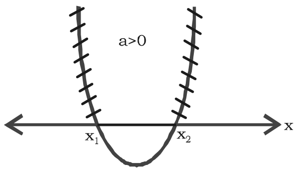
## 5. SOLUTION OF QUADRATIC INEQUALITIES

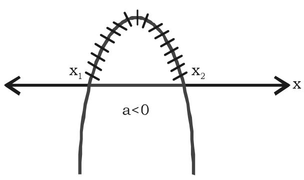
$ax^2 + bx + c > 0 \ (a \neq 0)$.

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $(x_1 < x_2)$.

Then $a > 0 \quad \Rightarrow \quad x \in (-\infty, x_1) \cup (x_2, \infty)$

$a < 0 \quad \Rightarrow \quad x \in (x_1, x_2)$



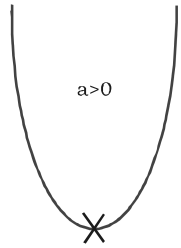

(ii) Inequalities of the form $\frac{P(x)}{Q(x)} \gtrless 0$ can be quickly solved using the method of intervals (wavy curve).

## 6. MAX. & MIN. VALUE OF QUADRATIC EXPRESSION

Maximum & Minimum Value of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as :

**For a > 0, we have :**

$$y \in \left[\frac{4ac - b^2}{4a}, \infty\right)$$



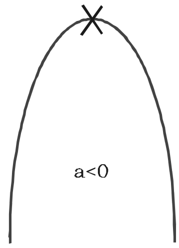

$$\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$$

$$y_{min} = \frac{-D}{4a} \text{ at } x = \frac{-b}{2a}, \text{ and } y_{max} \to \infty$$

**For a < 0, we have :**

$$\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$$

$$y \in \left(-\infty, \frac{4ac - b^2}{4a}\right]$$




$$y_{max} = \frac{-D}{4a} \text{ at } x = \frac{-b}{2a}, \text{ and } y_{min} \to -\infty$$

## 7. THEORY OF EQUATIONS

If $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are the roots of the $n^{th}$ degree polynomial equation :

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$$

where $a_0, a_1, \ldots a_n$ are all real & $a_0 \neq 0$,

Then,

$$\sum \alpha_1 = -\frac{a_1}{a_0};$$

$$\sum \alpha_1 \alpha_2 = \frac{a_2}{a_0};$$

$$\sum \alpha_1 \alpha_2 \alpha_3 = -\frac{a_3}{a_0};$$

............

$$\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

## 8. LOCATION OF ROOTS

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'k' are :

$D \geq 0$ & $f(k) > 0$ & $(-b/2a) > k$.

(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'k' (in other words the number 'k' lies between the roots of $f(x) = 0$ is:

$a\,f(k) < 0$.

(iii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval $(k_1, k_2)$ i.e. $k_1 < x < k_2$ are :

$D > 0$ & $f(k_1) . f(k_2) < 0$.

(iv) Conditions that both roots of $f(x) = 0$ to be confined between the numbers $k_1$ & $k_2$ are $(k_1 < k_2)$ :

$D \geq 0$ & $f(k_1) > 0$ & $f(k_2) > 0$ & $k_1 < (-b/2a) < k_2$.

*Note...*

**Remainder Theorem :** If $f(x)$ is a polynomial, then $f(h)$ is the remainder when $f(x)$ is divided by $x - h$.

**Factor theorem :** If $x = h$ is a root of equation $f(x) = 0$, then $x-h$ is a factor of $f(x)$ and conversely.

## 9. MAX. & MIN. VALUES OF RATIONAL EXPRESSION

Here we shall find the values attained by a rational expresion of the form $\frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2}$ for real values of x.

**Example No. 4** will make the method clear.

## 10. COMMON ROOTS

**(a) Only One Common Root**

Let $\alpha$ be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$, such that $a, a' \neq 0$ and $a b' \neq a'b$.

Then, the condition for one common root is :

$$(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c).$$

**(b) Two Common Roots**

Let $\alpha, \beta$ be the two common roots of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$, such that $a, a' \neq 0$.

Then, the condition for two common roots is :

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

## 11. RESOLUTION INTO TWO LINEAR FACTORS

The condition that a quadratic function

$f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

may be resolved into two linear factors is that ;

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

OR $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

## 12. FORMATION OF A POLYNOMIAL EQUATION

If $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are the roots of the $n^{th}$ degree polynomial equation, then the equation is

$x^n - S_1x^{n-1} + S_2x^{n-2} + S_3x^{n-3} + \ldots + (-1)^n S_n = 0$

where $S_k$ denotes the sum of the products of roots taken k at a time.

**Particular Cases**

**(a)** **Quadratic Equation** if $\alpha, \beta$ be the roots the quadratic equation, then the equation is :

$x^2 - S_1x + S_2 = 0$ *i.e.* $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

**(b)** **Cubic Equation** if $\alpha, \beta, \gamma$ be the roots the cubic equation, then the equation is :

$x^3 - S_1x^2 + S_2x - S_3 = 0$ *i.e.*

$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$

(i) If $\alpha$ is a root of equation $f(x) = 0$, the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$. In other words, $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If there be any two real numbers 'a' & 'b' such that f (a) & f (b) are of opposite signs, then f (x) = 0 must have atleast one real root between 'a' and 'b'.

(iv) Every equation f(x) = 0 of degree odd has atleast one real root of a sign opposite to that of its last term.

## 13. TRANSFORMATION OF EQUATIONS

(i) To obtain an equation whose roots are reciprocals of the roots of a given equation, it is obtained by replacing x by 1/x in the given equation

(ii) Transformation of an equation to another equation whose roots are negative of the roots of a given equation–replace x by – x.

(iii) Transformation of an equation to another equation whose roots are square of the roots of a given equation–replace x by $\sqrt{x}$ .

(iv) Transformation of an equation to another equation whose roots are cubes of the roots of a given equation–replace x by $x^{1/3}$.